EUREKA!

Archimedes' Principle

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Presented by Bichitra Pathshala (Learning with moving images)

This worksheet has activities and related questions based on Archimedes' Principle. It is advisable that the students try out at least some of the activities. The Archimedes' Principle has been chosen for the simple reason that the activities can safely be done with simple items of everyday use. Some activities are presented as thought experiments because it would not be practical to carry out these activities.

The Archimedes' Principle is usually introduced in class VIII and done in detail in class IX. This worksheet has been designed keeping the class IX students in mind. However, some calculations suggested in the worksheet may be tough for the class IX students. Students of higher classes may take these up. These questions are identified with an asterisk.

Activity 1: Buoyant force

A body immersed in water weighs less. If you ever went swimming you would know that. It feels lighter under water. And Archimedes' Principle tells us by how much.

Archimedes' Principle states that when a solid object is dipped in a fluid, it experiences a buoyant force equal to the weight of the fluid it displaces.

1. The king of Syracuse had told Archimedes to weigh an elephant. Archimedes devised a method using a big boat and lots of bricks which were all identical to each other. Can you guess why he used this method?

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Activity 2: Aluminium foil floats

Take a roll of aluminium foil and cut out a small piece of the size of, say, a one rupee coin. Place it gently on water with its surface horizontal. Watch it float. Now with one finger press it downward so that it is just under water. Leave it there. Watch it sink on its own to the bottom of the vessel.

Q 1: Given the fact that Aluminium has a density higher than that of water how would you explain what you observe? [Hint: surface tension plays a part.]

Now neatly cut out a few 10cm strips of aluminium foil. Carefully measure the width of one strip and calculate its area.

Now take one such strip and crumple it into a ball; press this ball below the upper surface of water. It floats back!

Q 2: Why does the ball float back while the flat foil sinks?

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Activity 3: Guess which side will weigh more

We start with a 'thought experiment'.
Take a good look at the picture on the side. C is a hollow cylinder; D is a solid one which fits exactly into C. Assume that the vessel with water is not there. We put D into C and balance it putting appropriate weights on the right hand side. Then we arrange things as shown in the picture – that is immerse D into water. Will the balance tilt? Which way?

Next we slowly fill C with water [up to the brim]; how will the balance shift this time?

Now on to the other experiments: we will go through each exercise first as a thought experiment, then verify it by actually doing it.

Take a glass bowl (or a tumbler or a small jug); fill it with water up to ¾th of the volume. Then get a piece of wood much smaller in size than the inner dimension of the vessel and proceed to do the following.

Step 1: Keep the water filled vessel and the wood side by side on the pan of the balance. Note the weight [W1].
Step 2: Let the wood float in the water of the vessel; note the weight [W2].

Q 1: Is W2 =, <, or > W1? Give reasons to support your answer.

Step 3: Remove the objects from the pan. Fill the vessel up to the brim. Weigh it [W3]. Remove it again and gently drop the wood in to the water. Water will overflow but in the vessel it will still be up to the brim; dry the outside of the vessel and carefully place it on the pan without spilling any more water. Note the weight [W4].

Q 2: Is W4 = , < , or > W3 ? Give reason to support your answer.

Activity 4: Cups and pins

Take a metallic cap of a bottle or one of those aluminium cups that hold small candles. Float it in water. Pour water into the cap little by little. You will see that the cap will remain afloat even when it is almost filled with water. Towards the end put water drop by drop till the cap sinks.

Can you guess how much water the cap can hold before it sinks?

Perhaps weighing the cap will help you get the answer. But how can you weigh such a small cap? You may go to the nearest goldsmith and ask him to weigh it in his electronic balance.

Do you find any correlation between the weight of the metallic cap and the volume of water it can hold before it sinks? It will not be very practical to find this co-relation empirically. So why not flex your mathematical muscles and try and find a mathematical relation?

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As long as the vessel floats, the upthrust or buoyant force is balanced by the total weight of the vessel and the water it holds.

1. Write an equation to express this equilibrium situation? Put the buoyant force on the left hand side of the equation and the total weight on the right hand side. What are the variables in this equation? What are the constants?
   [Hint: Take V as the inner volume of the vessel, (V + dV) as the outer volume of the vessel and D as the density of the material out of the which the vessel is made]

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***2. Supposing your vessel has a regular shape like a cylinder or a rectangular box. Can you rewrite the equation so that it has only one variable?

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***3. What is the maximum buoyant force the vessel can generate? What will be the expression of the right hand side of the equation when the left hand side acquires this maximum value? This equation will tell you about the maximum water the vessel can hold before it sinks.

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Now repeat the same experiment with a small plastic vessel – say, a small plastic cap that comes with the medicine bottles. Float this vessel in water and then slowly pour water in the vessel till it is full to the brim. Why does this vessel not sink even when filled to the brim? To get an answer, go back to the equation you had written earlier. What are the parameters that have changed in the case of plastic? How is the situation different as a result of this changed parameter?

In order to sink this floating plastic vessel filled to the brim with water, you put pins into the vessel one by one. Count the number of pins it can hold before it sinks completely.

Repeat this experiment carefully a number of times to see if the number of pins comes the same each time. Once you are satisfied, dry the pins and weigh them. You may again have to go to a goldsmith to get an accurate weight of the pins.

4. Rewrite the equation incorporating the weight of the pins [n x p – where n is the number of pins and p is the weight of each pin]. What will this weight be equivalent to?

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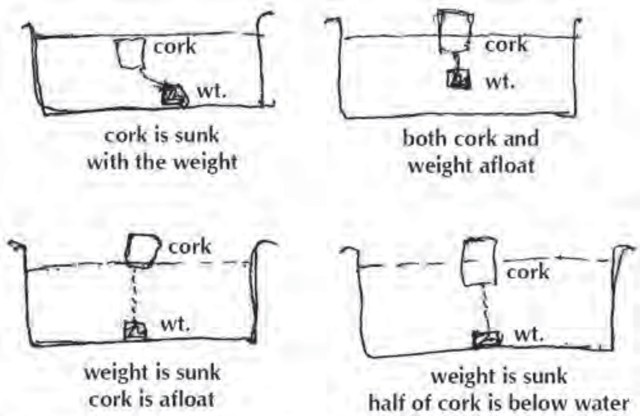
Activity 5: Tension between floating and sinking



Guess the tension in each case

Take a piece of cork or a piece of thermocol. Screw a hook to the bottom. Tie a short thread to the hook. At the other end of the thread tie the smallest weight in your weight box. Put this system in water and see if it sinks.

If it floats, then take out the smallest weight and tie the bigger weight in your weight box.
Keep repeating this experiment and tying bigger and bigger weights to the thread till the cork and the weight sink.

Q 1: Can you get a rough estimate of the volume of the cork or thermocol piece from this experiment?

Q 2: What will be the tension in the thread when both the cork and the weight remain afloat?

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Q 3: What will be the tension in the thread when both of them sink?

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Q 4: What will be the tension in the thread when the thread is long enough so that even when the weight sinks to the bottom of the container, the cork remains afloat?

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Q 5: What will be the tension in the string when the length of the string is such that half the cork remains afloat above water? (When the cork floats on its own, practically the entire volume of the cork is above water)

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Activity 6: Ice and water

All of you must have frozen ice in an ice tray and all of you know that ice floats in water. Perhaps you also know that when an ice cube floats in water, one twelfth of its volume remains above the surface of the water and 11/12th remains below the surface. This is because the density of ice is 11/12th that of water at 4 degrees centigrade.

Now freeze some ice with a difference. Before pouring water into the ice tray, put some needles or pins in the cubes. Put one needle in one cube, two needles in the next, three needles in the one after that and so on. Make sure that at least in one cube there is no needle. Keep a track of the number of needles you have put in each cube. Now fill all the cubes up to the brim with water and put in the ice chamber of your fridge. Leave it like that for a few hours for the ice to form. Then take out the ice cubes one by one and put them on water one by one to see if they remain afloat or sink. As you go on putting ice cubes with more and more number of pins in the water, they become heavier and heavier and will immerse more and more in water. Eventually one ice cube will sink. How many pins does this ice cube have inside it?

1. Can you find the weight of one pin from this experiment? What else will you require to find the weight of the pin? Outline a step by step method of finding the weight of one pin from this experiment. Then verify your result by weighing the pins on a goldsmith's weighing machine or in the weighing machine in your chemistry lab.

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   Next day again freeze ice by putting different numbers of pins or nails in different cubicles of the ice tray. But now you know which ice cubes will sink and which will remain afloat. After the ice is formed, put the ice cubes that you expect to remain afloat in one tumbler and the ice cubes that you expect to sink in another tumbler. Using a marker pen mark the level of water in both the tumblers.

2. If you take identical tumblers with exactly the same level of water initially, in which tumbler will the level of water rise more – the one in which the ice sinks or the one in which the ice floats?

3. Now put both the tumblers aside and wait for the ice to melt. Check the level of water in both the tumblers after the ice melts. Of course, once the ice melts, the pins will fall to the bottom of the tumbler. Does the level of water remain the same in the tumblers or does it change? What explanation do you have for what you observe?

If you are still in a mood to play with ice, try out the following:

Take a medium sized candle and cut out cross sections that are roughly half a centimeter thick. Now take the ice tray and put one piece of candle in one cubicle, two pieces in another one, three in the next one and perhaps four in the fourth one and repeat the pattern for the rest of the cubicles. Then pour water upto the brim and put it in the freezer for ice to form.

4. Do you expect the ice to float or sink?

If possible, put all the pure ice cubes with no wax in one tumbler, ice cubes with one piece of wax in the second tumbler, ice cubes with two pieces of wax in the third tumbler and so on.

5. In which tumbler do you observe the level of water to rise the highest?

6. What happens to the level of water in all the tumblers when the ice melts?

If possible, use a video camera to film the process and share it with your friends.

Activity 7: Make your own Dead Sea

We're sure, you have read about the Dead Sea in your Physics text books. It is a land locked sea in Israel where the water is so salty that you can lie down on the water without the fear of being drowned.

Create your own Dead Sea in a tumbler. Fill the tumbler with tap water and put a raw egg in the water. The egg will sink. Now add salt to the water in small pinches and keep stirring gently to dissolve the salt. While stirring don't hit against the egg – or it will break and there will be a mess.

As you keep adding salt in pinches, you will see that the egg will begin to float up at one point of time. Add a little more salt and safely the egg will remain afloat in water.

1. Now the question is, why does the egg float up in salty water and why do we remain afloat in the Dead Sea.

2. Try and see what other things that sink in tap water remain afloat in salty water. Do you think nails and pins will remain afloat in salty water? If you can get hold of anything besides an egg that will remain afloat in salty water, but sink in tap water, take its picture and send it to your friends and to us. When you are trying various objects, do not forget to try out lumps of clay and plasticine.

3. When you go out to swim in the swimming pool, the water is definitely not salty. So you do not automatically remain afloat, although you do feel the buoyant force. You do feel much lighter in water. After all Archimedes had discovered about buoyant force when he was having a bath in the public bath. Your swimming instructor must have also told you that when you take a deep breath and fill your lungs with air and then pinch your nose and dive under water, you will have less chances of drowning. On the other hand, while drowning if you gulp down water, you have greater chances of drowning. Explain why.

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4. Lastly, why does a dead body float up in water?

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Activity 8: Find out how fishes and submarines float and sink at will

When the weight of water displaced by a body exceeds its own weight, the body floats. Otherwise it sinks. But a fish can float or sink at its will! So can a submarine. Not only that, they are able to settle or hover at any depth they desire. How do they do it? This activity will tell you how.

Before starting it let us acquaint ourselves with some essential facts. Most fishes have swim bladders [internal air bags] and they can alter the amount of gas in the bladder. In a submarine this task is accomplished by altering the amount of water [and hence the amount of air] in its 'ballast' tanks.

Now, we mimic this with a very simple device – an inverted glass tube. Take a narrow test tube or a (typical) glass vial in which homeopathic medicines are kept. Wind some metal wire around its open end to make that end heavier than the other. [Soldering wire is very suitable for this – but any other metal wire will serve the purpose.]

Now gently drop this thing into a bucket of water.

Q 1: What would/do you see?

Q 2: Can you explain what you observe keeping in mind the fact that glass is heavier [i.e. has a higher density] than water and so is the metal?

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Now take an empty 2-litre bottle of mineral water. Make sure that the cap makes it air-tight. Fill it with water up to the brim. Gently drop the device [also known as 'the Cartesian Diver'] with the wired side down. Adjust the length of the wire to ensure that the device just sinks with its upper end touching the surface of water. Close the cap. Now gently squeeze the bottle.

Q 3: How is the position and movement of the 'diver' related to the degree of squeezing?

Q 4: Can you settle it at any depth you desire?

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Q 5: Observing carefully the trapped air inside the 'diver' can you now explain how fishes and submarines do what they do?

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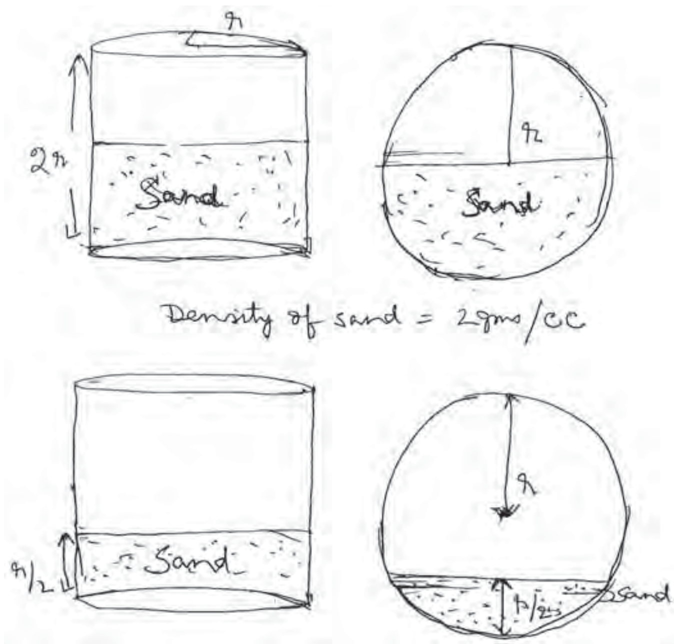
Activity 9: Archimedes also worked on geometry

Archimedes' tomb in Syracuse has the inscription of a cylinder with a sphere inscribed in the cylinder. This means that the sphere is touching the cylinder on the two sides as well as at the top and bottom. The volume of such a sphere is two-thirds that of the cylinder. The surface area of the sphere is also two-thirds that of the cylinder if we add the areas of the top and the bottom to the area of the curved surface of the cylinder. You can verify this mathematically by applying the formulae given at the end of the question.



Archimedes had said that this was the greatest discovery of his life.

Now let us do some thought experiments with these solids – the sphere and the cylinder. Suppose you are given an Archimedes cylinder of radius 'r' and height '2r' and also an Archimedes' sphere of radius 'r'. Both the cylinder and the sphere are hollow and made of hard, transparent, weightless material. The thickness of their walls is practically zero. These are idealized conditions no doubt, but then you are doing thought experiments and not real experiments.

Needless to say, both the sphere and the cylinder will float in water. Both the solids have a small hole at the top, through which you can pour a liquid or a powdery solid. You are given dry sand whose density is 2gms/cc, that is, twice the density of water. This is again idealized sand. For all practical purposes you will not find sand of such a description.

Q 1: Guess what will happen to the sphere and the cylinder when you pour sand in both the solids so that both are half full with the sand. Draw a diagram to show the positions of the two solids when both are half filled with the sand given to you. Support your inference with calculations.

Q 2: Now imagine that you pour out the sand from both the solids and pour the sphere's sand into the cylinder and vice versa. Guess what will happen when the sand in the two solids are interchanged. Again draw a diagram to show the positions of the two solids. Support your inference with calculations.

Q 3: Suppose the sphere and the cylinder are immersed in a cylindrical jar of water whose radius is '5r. Calculate the rise in the level of water in the jar both for questions 1 and 2.

*****Q 4 (A real tough one)**: Suppose you fill the cylinder and the sphere up to a quarter of their respective heights, that is, up to a height of 'r/2'. Guess their respective positions in water.

Q 5: You fill the glass jar up to the brim and then fully immerse the cylinder in the water, either by pushing it down with a needle or by filling it with sand. Collect the overflowing water in a small beaker without spilling any water. Take out the cylinder and do the same thing with the sphere. If you collect the overflowing water in both the cases in identical beakers without spilling any water in either case, what will be the ratio of the heights of water in the two beakers?

Q 6: Now suppose that you do not have two identical beakers. One of the beakers is narrower than the other. After collecting the overflowing water in these two beakers, you find that the level of water is the same in both the beakers. Can you calculate the ratio of their respective radii or the ratio of their respective areas of cross sections?

If instead of water, the solids are immersed in oil and the overflowing oil is collected in the same beakers. Will the levels of the oil in the two beakers be the same as before or will they be different?

In order to solve the above questions you need to know the formulae for volumes and surface areas of spheres and cylinders:
Volume of sphere: 4/3 x 3.14 x r x r x r
Surface area of sphere: 4 x 3.14 r x r

Volume of cylinder: 3.14 x r x r x h
Surface area of cylinder: 2 x 3.14 x r x h

('x' indicates the symbol for multiplication, 'r' is radius and 'h' stands for height)

Activity 10: Your own hydrometer

The instrument in the picture on the left is called a **Hydrometer**. You know 'hydro-'relates to water and a meter must measure something. A Hydrometer is used to measure the specific gravity of water and other liquids. Look at the gradation on its arm [reminds you of a thermometer perhaps] – you can read the value of the specific gravity [and thereby determine the density] of a liquid by noting the line which is just above the water/other liquid. This was discovered long time ago [we will come to that later] – but is still used almost in an unaltered form. When it is designed specifically to measure density of Milk it is called a **Lactometer**. Similarly you can have a **Battery hydrometer** to test the specific gravity of acid in a car battery, on the right, above, you see two of them. But the basic working principle is the same for all of them.

Now – into Action! We build our own Hydrometer.
All you need is a long and narrow body that floats in water and some metal wires; as in *Activity 8*, Soldering wire works best.



You could take a pencil or similarly shaped plastic or wooden cylinder. The most suitable object we found was an empty plastic tube of 'whitener' – the white ink that we use (literally) to cover up our mistakes! Wind just enough wire on to its fat end so that it floats erectly in water with about 1/3rd or 1/4th of it above water. [Have a look at the photo at the middle and right.]

Drop your hydrometer in water and wait till it is stable. Put a round mark along the water meniscus touching it. Recall that specific gravity of water is one.

1: Using your hydrometer what can you say about the specific gravities of cooking oil, kerosene oil, salt water and milk [with your Mother's permission]?
Try it on other liquids around you and note the results. [Don't try it on acids unless your parent/teacher is there to help you].

Now go and check the specific gravities of 5 other liquids and record them in two groups: heavier and lighter than water.

Archimedes' Principle (Answers)

Activity 1: Archimedes suggested the following:
Put the elephant on the boat and mark the level of water. Then remove the elephant and put bricks till the boat sinks to the same level. Count the bricks in the boat. Measure the weight of one brick.

Activity 2
Q 2: The crumpled ball floats back because it is able to displace more water [due to air trapped between layers of foil] and hence generate more buoyant force.

Activity 3
Q 1: W2 = W1. (pressure exerted by wood on the bottom of the vessel equals buoyancy.)
Q 2: W4 = W3 [weight of displaced water equals that of the wood.]

Activity 4
1. As long as the vessel floats, buoyant force is equal to the weight of the vessel plus the weight of the water in the vessel. Therefore
v x 1 = (v^ x 1) + (dV x D) where v is the volume of the vessel under water and v^ is the volume of water in the vessel. These are the two variables in the equation.

2. Now for a cylinder or cube or rectangular parallelepiped, Volume = Area of base X height.
If the vessel is under water up to a depth h, then volume of the water displaced is A x h
If the level of water in the vessel is up to a height s, volume of water in the vessel is A x s

So the above equation may be rewritten as:
A x h = (A x s) + (dV x D) (dV x D) may be replaced by W, the weight of the vessel.
A x h = (A x s) + W
'h' and 's' are the two variables. The relation between 'h' and 's' is:
(h – s) = W/A This is a constant.

3. The maximum buoyant force the vessel can generate is (V + dV) x 1, where V is the inner volume of the vessel.
This will be balanced by the weight of the vessel plus the weight of water inside the vessel. Therefore

(V + dV) x 1 = (V^ x 1) + (dV x D) where V^ is maximum volume of water that the vessel can hold.

For a cylinder or cube or a rectangular parallelepiped or similar shaped vessels, one may rewrite the above equation in terms of height of water:

A x H = (A x S) + W where H is the height of the vessel and S is the height of the water in the vessel. S, which is the only unknown, may be found from this equation:
S = [(A x H) – W]/ A = H – W/A

4. When you are repeating the above experiment with a plastic vessel, you have to take the density of plastic into account, which is less than 1gm/cc. Therefore the equation in Ans 3 will be slightly modified:

V* x 1 = (V x 1) + (dV x D) where V* is the volume of the vessel under water when the vessel is full upto the brim. When the vessel is just on the verge of sinking with n pins, this equation will be re written as:

(V + dV) x 1 = (V x 1) + (dV x D) + (n x p)
Or
dV = (dV x D) + (n x p)
Or
(n x p) = dV(1 – D)

Activity 5
Q 2: Tension = 0 [wt – volume of the wt x 1]
Q 3: Tension = (Volume of cork x 1) – Weight of cork
Q 4: Tension = 0
Q 5: Tension = (Volume of cork x 1)/2 – Weight of cork

Activity 6
1: Suppose the ice cube with N pins just sinks in water. Suppose weight of one pin is p. Then
N x p = Volume of ice cube/12

Activity 7
3: While the human body (once you fill your lungs with air) is just dense enough to manage to float in ordinary water, keeping your nose out of water and breathing normally requires effort. This is because the head is somewhat denser than the rest of the body and thus sinks. [Average specific gravity of the body is almost 1, while that of the head is 1.1 approximately.] So you must paddle with your feet while floating, although fairly gently, to keep your nose out of water.

4: Now, dead-bodies initially sink, usually head first. After a few days, they float back to the surface due to decomposition which generates gases [inflating the body]. If left in the water, the tissue breaks down enough to release the trapped gases, so it sinks again.

Activity 8
Q 1 & 2: Some amount of air will be trapped in the upper portion of the tube. Volume of the displaced water is the sum of the volumes of wire + glass [under water] + air so that buoyancy is enough keep the system afloat.
Q 4: Yes.
Q 5: In our case volume of trapped air decreases with increasing outside pressure. Fishes adjust the volume of the swim bladder; submarines alter the amount of water in the ballast tanks.

Activity 9
Q 1: Both the sphere and the cylinder will be fully submerged in water
Q 2: When the sands are interchanged, the sphere will sink and the cylinder will be 4r/3 under water
Q 3: When the cylinder is fully immersed, rise in the level of water in the jar is 2xR/25
When the sphere is fully immersed in water, rise in the level of water in the jar is 4xR/75
Q 4: Cylinder will be half under water. 5/16 of the volume of the sphere will be under water
Q 5: Ratio of the heights in the beaker will be 3:2
Q 6: Ratio of the radii is square root of 3/2

Activity 10
1: Specific gravity of oils less than one; that of salt water and milk more than one.